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Institute of Mathematical Sciences
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RESEARCH REPORT No. EM-162

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Abstract

When a wave of wavelength λ is incident upon an opaque object of typical dimension a, a shadow is formed in the geometric optics limit $\lambda/a = 0$. If λ/a is small and not zero, the shadow boundary is displaced slightly from the geometrical shadow boundary. It is known from the work of Artmann that the amount of the shift in the case of circular cylinders is asymptotically equal to $\alpha(\lambda^2 a)^{1/3}$. The proportionality constant α may be positive or negative, depending on the boundary conditions satisfied by the field. A similar result was obtained for parabolic cylinders by Rice.

Because of the discrepancy in the proportionality constant for hard cylinders that exists between the works of Rice and Artmann, already pointed out by Wait and Conda, we have redetermined the shift for the circular cylinder and found that α was the same as for the parabolic cylinder as determined by Rice. In addition, the shift has been determined for circular cylinders for fields which satisfy an impedance boundary condition.

The agreement of the results for circular cylinders and parabolic cylinders can be understood in terms of the local nature of shadow boundary rays. On this basis we conjecture that the shift of the shadow boundary cast by any two or three dimensional object when λ/a is small is asymptotically equal to $\alpha(\lambda^2 a)^{1/3}$ along each boundary ray. The constant α depends upon the impedance of the object at the point of tangency and is given by our result for the circular cylinder. This conjecture is consistent for waves at oblique incidence.

We have also determined the scattering cross section per unit length of a circular cylinder for λ/a small and for an impedance boundary condition. We find that the deviation from the geometrical optics cross section is proportional to the shift of the shadow boundary. This, together with the local character of the

of the shift, leads us to conjecture that the total scattering cross section for any two or three dimensional object when λ/a is small is asymptotically equal to the integral around a cross section of the shadow boundary of a multiple of the shift. This conjecture is checked by applying it to spheres and showing that it yields the known exact results. Finally, we show how similar results and conjectures may be applied to the case of incident electromagnetic waves.

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1. Introduction.

When a wave is incident upon an opaque object of typical dimension a, which is large compared to the incident wavelength λ, a shadow is formed. In the geometrical optics limit, $\lambda/a = 0$, the shadow is bounded by those optical rays which are tangent to the object. However when λ/a is small and not zero, the shadow boundary is parallel to, but displaced slightly from, the geometrical shadow boundary. This fact was first discovered theoretically by K. Artmann in 1950 [1]. He considered a scalar plane wave normally incident upon a circular cylinder of radius a and showed that the shift was asymptotically equal to $\alpha(\lambda^2 a)^{1/3}$. The proportionality constant α was found to be positive for a soft cylinder, on which the field vanishes, and negative for a hard one, on which its normal derivative vanishes. Thus in the former case the shadow was enlarged while in the latter case it was diminished. A similar result was later obtained by S.O. Rice [2] for the shadow cast by a parabolic cylinder. In this case a was the radius of curvature of the cylinder at the point of tangency of the geometrical shadow boundary.

Since fields propagate along rays when λ/a is small, we then expect each ray forming a shadow boundary to be shifted indepently of other such rays. In other words, we expect the shadow boundary shift to be a local effect. If so, the shift of each shadow boundary ray should depend only upon the curvature of the object at the point where the ray is tangent to it. This expectation is partly confirmed by the fact that Artmann obtained nearly the same value for the constant α for a soft circular cylinder as Rice did for a soft parabolic cylinder. However they obtained unequal values for hard cylinders. Since this result is contrary to our expectation, and since Rice checked his work without

finding an error, we redetermined the shift for the circular cylinder and found that α was the same as for the parabolic cylinder in both cases. Thus the expectation that the shift is a local effect is confirmed. In addition we have determined the shift for circular cylinders on which the field satisfies an impedance boundary condition.

This confirmation leads us to conjecture that the shift of the boundary of the shadow cast by any two or three dimensional object when λ/a is small is asymptotically equal to $\alpha(\lambda^2 a)^{1/3}$ along each boundary ray (see equation (27)). Here a denotes the radius of curvature of the surface of the object at the point of tangency of the ray, in a plane containing the ray and the normal to the surface. The constant α depends upon the impedance of the object at the point of tangency and is given by our result for the circular cylinder. To check our conjecture we show that it is consistent for a plane wave obliquely incident upon a cylinder of arbitrary cross section. This means that it is true for oblique incidence if it is true for normal incidence. Since it is true for normal incidence in the cases of a circular cylinder of any impedance and of soft and hard parabolic cylinders, it is also true for oblique incidence in those cases.

Next we determine for λ/a small, the total scattering cross section per unit length of a circular cylinder on which the field satisfies an impedance boundary condition. We find that the deviation from the geometrical optics cross section is proportional to the shift of the shadow boundary. This proportionality, and the local character of the shift, suggest a conjecture for the total scattering cross section of any two or three dimensional object when λ/a is small. It is that the deviation from the geometrical optics cross section is asymptotically equal to the integral, around a cross section of the shadow boundary, of a multiple of the shift. The multiplier is just the proportionality

factor found for the circular cylinder. This conjecture is checked by applying it to hard and soft spheres and showing that it yields the known exact results.

Finally we consider electromagnetic waves incident upon perfectly conducting objects. We determine the shadow boundary shift and the scattering cross section for a circular cylinder. We obtain results similar to those we obtained in the scalar cases and make similar conjectures for arbitrary objects.

The field near the boundary of the shadow of a circular cylinder has also been determined by J.R. Wait and A.M. Conda [3] when the field satisfies an impedance boundary condition on the cylinder. In the cases $Z=0$ and $Z=\infty$ they have shown that the value of the field on the geometrical optics boundary agrees with that found by S.O. Rice [2] on the geometrical optics boundary of the shadow of a parabolic cylinder. They have also pointed out that the corresponding results of K. Artmann [1] differ from theirs and Rice's, and attribute this difference to an inaccuracy in his calculation.

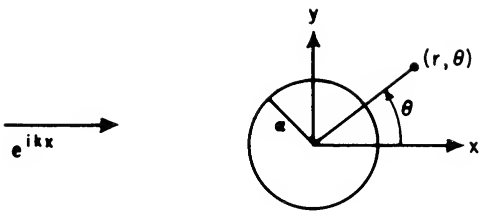

Figure 1 - A plane wave e^{ikx} is incident from the left upon a circular cylinder of radius a.

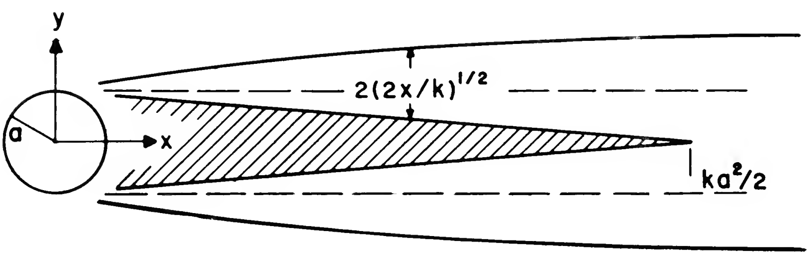

Figure 2 - The shadow cast by a circular cylinder illuminated from the left by a plane wave. The dashed lines are the geometrical shadow boundaries. The solid lines bound the transition zones between the illuminated and shaded regions. The width of each zone is approximately $2(2x/k)^{1/2}$ at a distance x behind the cylinder. The two zones merge at the (approximate) distance $ka^2/2$ which determines the length of the shadow.

2. The scattered field.

We wish to determine the field $u(r,\theta)$ which results when a plane wave e^{ikx} is incident from the left upon a circular cylinder of radius a. (See Figure 1.) The field satisfies the reduced wave equation

$$(\Delta + k^2)u = 0, \qquad r \geq a \tag{1}$$

On the cylinder it satisfies the impedance boundary condition

$$\Omega u \equiv \left(\frac{\partial u}{\partial kr} + iZu\right) = 0 \text{ at } r = a \tag{2}$$

Here Z is a constant called the impedance of the cylinder surface and Ω is the boundary operator. The scattered field $u_s(r,\theta)$, defined by $u = e^{ikx} + u_s$, must satisfy the radiation condition

$$\lim_{r \to \infty} r^{1/2}\left(\frac{\partial u_s}{\partial r} - iku_s\right) = 0 \tag{3}$$

Equations (1) - (3) have a unique solution u for which u_s is given by

$$u_s(r,\theta) = -\frac{1}{2}\sum_{n=-\infty}^{\infty} e^{in(\frac{\pi}{2}+\theta)} H_n^{(1)}(kr)\left[1 + \frac{\Omega H_n^{(2)}(ka)}{\Omega H_n^{(1)}(ka)}\right] \tag{4}$$

We can rewrite (4) in another form by introducing $F(r,\theta)$, the Fourier transform of the summand, defined by

$$F(r,\theta) = -\frac{1}{2}\int_{-\infty}^{\infty} e^{i\nu(\frac{\pi}{2}+\theta)} H_\nu^{(1)}(kr)\left[1 + \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\right] d\nu \tag{5}$$

In terms of F, the Poisson summation formula yields for (4),

$$u_s(r,\theta) = \sum_{m=-\infty}^{\infty} F(r, 2\pi m+\theta) \tag{6}$$

We now rewrite the terms in (6) for which m is negative by replacing m by -m'. Then upon changing ν to $-\nu'$ in (5) and making use of the relations between Hankel functions of positive and negative orders, we finally obtain

$$u_s(r,\theta) = G(r,\theta) + G(r,-\theta) + \sum_{m=1}^{\infty} \left[F(r,2\pi m+\theta) + F(r,2\pi m-\theta)\right] \tag{7}$$

Here $G(r,\theta)$ is defined by

$$G(r,\theta) = -\frac{1}{2}\int_{-\infty}^{\infty} e^{i\nu(\frac{\pi}{2}+\theta)} H_\nu^{(1)}(kr)\left[h(\nu) + \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\right] d\nu \tag{8}$$

In (8) $h(\nu)$ is the unit step function defined by

$$h(\nu) = 0,\ \nu < 0;\ h(\nu) = 1,\ \nu > 0 \tag{9}$$

From u_s we can compute σ, the scattering cross section per unit length of cylinder. If Z is real the cross section theorem yields

$$\sigma = \lim_{r\to\infty} \text{I.P.}\left[\left(\frac{8\pi r}{k}\right)^{1/2} e^{-i(kr+\pi/4)} u_s(r,0)\right]$$

$$= \frac{4}{k}\,\text{R.P.}\int_{-\infty}^{\infty}\left[h(\nu) + \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\right] d\nu + \frac{4}{k}\,\text{R.P.}\sum_{m=1}^{\infty}\int_{-\infty}^{\infty} e^{2\pi m\nu}\left[1 + \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\right] d\nu \tag{10}$$

When Z is complex σ denotes the extinction cross section which is the sum of the scattering and absorption cross sections.

3. <u>Asymptotic form of u near the shadow boundary.</u>

The term $F(2\pi m+\theta)$ in (7) represents a diffracted or creeping wave which has traveled m times around the cylinder in the direction of increasing θ. Similarly $F(2\pi m-\theta)$ represents a wave which has traveled m times around

in the direction of decreasing θ. These diffracted waves decay exponentially with increasing angle around the cylinder with a decay rate proportional to $(ka)^{1/3}$. Therefore for large values of ka, which means for a large compared to the wavelength $\lambda = 2\pi/k$, all the terms in the sum in (7) are exponentially small compared to G. Consequently in evaluating u_s for ka large, we need only consider the terms $G(r,\theta)$ and $G(r,-\theta)$.

We shall find it convenient to split the integral in (8) into two terms $G_1(r,\theta)$ and $G_2(r,\theta)$ defined by

$$G_1(r,\theta) = -\frac{1}{2}\int_0^{ka} e^{i\nu(\frac{\pi}{2}+\theta)} H_\nu^{(1)}(kr)d\nu \ , \tag{11}$$

$$G_2(r,\theta) = -\frac{1}{2}\int_{-\infty}^{ka} e^{i\nu(\frac{\pi}{2}+\theta)} H_\nu^{(1)}(kr)\,\frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\;d\nu$$

$$-\frac{1}{2}\int_{ka}^{\infty} e^{i\nu(\frac{\pi}{2}+\theta)} H_\nu^{(1)}(kr)\left[1+\frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)}\right]d\nu \tag{12}$$

Then

$$G(r,\theta) = G_1(r,\theta) + G_2(r,\theta) \tag{13}$$

To evaluate G_1 asymptotically for large ka and $r > a$ we may replace $H_\nu^{(1)}(kr)$ by its Debye asymptotic form which is valid when kr is large and $kr > \nu$. We also introduce the integration variable $\tau = \sin^{-1}(\nu/kr)$. Then (11) becomes

$$G_1(r,\theta) \sim -e^{-i\pi/4}\left(\frac{kr}{2\pi}\right)^{1/2} \int_0^{\sin^{-1}(a/r)} (\cos\tau)^{1/2} \exp\left\{ikr\left[\cos\tau + (\theta+\tau)\sin\tau\right]\right\} d\tau \tag{14}$$

Let us now evaluate $G(r,-\theta)$ on and near the upper boundary of the shadow cast by the cylinder. Since ka is large and $r > a$, kr is also large so we may employ the method of stationary phase to evaluate $G_1(r,-\theta)$. The stationary phase point is at $\tau=\theta$ or, in terms of the cartesian coordinate y, at $\tau=\sin^{-1}(y/r)$. Near the shadow boundary $y=a$ this stationary point is near the upper limit of integration. This must be taken into account in evaluating the integral asymptotically. By expanding the phase up to quadratic terms around the stationary point we obtain

$$G_1(r,-\theta) \sim -e^{-i\pi/4}\left(\frac{kr}{2\pi}\right)^{1/2}\cdot\left(\frac{x}{r}\right)^{1/2} e^{ikx} \int_0^{\sin^{-1}(a/r)} e^{ikx(\tau-\theta)^2/2}\, d\tau \tag{15}$$

Now we introduce $t = (\tau-\theta)(kx/2)^{1/2}$ as integration variable and (15) becomes

$$G_1(r,-\theta) \sim -e^{i(kx-\pi/4)}\pi^{-1/2} \int_{-(kx/2)^{1/2}\theta}^{(kx/2)^{1/2}\left[\sin^{-1}(a/r)-\theta\right]} e^{it^2}\, dt \tag{16}$$

If the lower limit of the integral in (16) is large in absolute value and the upper limit small in absolute value, (16) yields

$$G_1(r,-\theta) \sim -\frac{e^{ikx}}{2}\left[1 + e^{-i\pi/4}\left(\frac{2kx}{\pi}\right)^{1/2}\left\{\sin^{-1}(a/r)-\theta\right\}\right]$$

$$\sim -\frac{e^{ikx}}{2}\left[1 + e^{-i\pi/4}\left(\frac{2k}{\pi x}\right)^{1/2}(a-y)\right] \tag{17}$$

The conditions under which (17) holds, which were imposed upon the integration limits in (16), are

$$(\frac{kx}{2})^{1/2} \tan^{-1}(a/x) \gg 1 \quad , \tag{18}$$

$$(\frac{k}{2x})^{1/2} |a-y| \ll 1 \quad . \tag{19}$$

Equation (18) is equivalent to $x \ll ka^2/2$, which means that x must be small compared to the length of the shadow $ka^2/2$. Equation (19) requires that the distance $|a-y|$ of the point from the geometrical shadow boundary y=a be small compared to the half width $(2x/k)^{1/2}$ of the shadow boundary zone. (See Figure 2).

To evaluate $G_2(r,-\theta)$ asymptotically for large ka we first insert into the integral the Debye asymptotic form for the Hankel functions. This asymptotic form is valid only when the order and argument are unequal. Therefore it is not valid for one of the Hankel functions when ν=kr and for the other two when ν=ka. However the neighborhoods of these two points contribute asymptotically negligible amounts to the integral if the point r,θ does not lie on or near the upper boundary of the shadow. This can be seen by noting that at all other points in the illuminated region for which $0 < \theta < \pi$, the integral can be evaluated asymptotically by the method of stationary phase. The stationary point, whose neighborhood yields the entire asymptotic contribution to the integral, is away from ν=kr and ν=ka. In the lower half of the (r,θ) plane, the entire integral is asymptotically negligible compared to $G_2(r,\theta)$. However when the point r,θ approaches the upper boundary of the shadow, the stationary point approaches ν=ka. Therefore we conclude that for points on and near this boundary, the entire asymptotic contribution to the integral comes from the neighborhood of ν=ka. But then the Debye asymptotic forms of $H_\nu^{(1)}(ka)$ and $H_\nu^{(2)}(ka)$ cannot be used since they are not valid at ν=ka. Instead we must use their uniform asymptotic

representations in terms of the Airy function A. For $H_\nu^{(1)}(kr)$ we may still use the simpler Debye asymptotic form provided that $r > a$, and expand it around $\nu = ka$. In the special case of forward scattering, for which $\theta = 0$ and $r \gg a$, it has been proved that this procedure does yield the asymptotic form of the integral [4].

In order to carry out the procedure just described for points on and near the upper shadow boundary we first consider the Debye asymptotic form of $H_\nu^{(1)}(kr)$. In it we replace ν by the new variable q by means of the equation

$$\nu = ka + \left(\frac{ka}{6}\right)^{1/3} q \tag{20}$$

Then we multiply it by $e^{i\nu(\frac{\pi}{2} - \theta)}$ and simplify the product by considering y-a to be small and x to be positive. In this way we obtain

$$e^{i\nu(\frac{\pi}{2} - \theta)} H_\nu^{(1)}(kr) \sim e^{i(kx - i\pi/4)} \left(\frac{2}{\pi kx}\right)^{1/2} \tag{21}$$

Now we insert (21) and the uniform asymptotic forms of the other Hankel functions into (12) and introduce q as the integration variable. Then we obtain in the neighborhood of the upper shadow boundary

$$G_2(r,-\theta) \sim -(2\pi kx)^{-1/2} e^{i(kx - \frac{\pi}{4})} (ka)^{1/3} C\left[(ka)^{1/3} Z\right] . \tag{22}$$

Here C is defined by

$$C\left[(ka)^{1/3} Z\right] = 6^{-1/3} \int_{-\infty}^{0} f\left[q, (ka)^{1/3} Z\right] dq + 6^{-1/3} \int_{0}^{\infty} \left\{ 1 + f\left[q, (ka)^{1/3} Z\right] \right\} dq \tag{23}$$

where

$$f\left[q,(ka)^{1/3}Z\right] = e^{i\frac{4}{3}\pi}\,\frac{A'(e^{i\frac{\pi}{3}}q) - ie^{-i\frac{\pi}{3}}(ka/6)^{1/3}ZA(e^{i\frac{\pi}{3}}q)}{A'(e^{-i\frac{\pi}{3}}q) - ie^{i\frac{\pi}{3}}(ka/6)^{1/3}ZA(e^{-i\frac{\pi}{3}}q)}\;. \tag{24}$$

The Airy function in (24) is defined by

$$A(q) = \frac{1}{2}\int_{-\infty}^{\infty} \exp\left[-i(\xi^3-q\xi)\right]d\xi \tag{25}$$

For points at which the conditions (18) and (19) are satisfied, $G_2(r,\theta)$ is asymptotically negligible compared with $G_2(r,-\theta)$. Physically this is so because $G_2(r,\theta)$ describes the reflected wave in the region $0 > \theta > -\pi$ and describes a diffracted wave in the region $0 < \theta < \pi$. The diffracted wave decays exponentially with increasing θ and it is exponentially small compared to $G_2(r,-\theta)$ near the upper shadow boundary, provided (18) and (19) are satisfied. Thus when these conditions are satisfied the field on and near the upper shadow boundary, $u = e^{ikx} + u_s$, is from (7), (13), (17) and (22)

$$u \sim \frac{e^{ikx}}{2} - (2\pi kx)^{-1/2}e^{i(kx-\pi/4)}\left\{k(a-y)+(ka)^{1/3}C\left[(ka)^{1/3}Z\right]\right\}\;. \tag{26}$$

4. <u>The displacement of the shadow boundary.</u>

From (26) we see that at the geometrical shadow boundary y=a, u approaches $e^{ikx}/2$ as ka becomes infinite. Thus the field on the geometrical shadow boundary approaches one half the incident field in the geometrical optics limit. This suggests that we define the shadow boundary for finite values of ka as the curve on which $|u| = 1/2$. By applying this definition to (26) we find that the shadow boundary is then the straight line $y = a+s$ where s is given by

$$s = (C_R+C_I)(ka)^{-2/3}a \tag{27}$$

Here C_R and C_I are the real and imaginary parts of $C\left[(ka)^{1/3}Z\right]$. Thus the shadow boundary is displaced or shifted from the geometrical boundary y=a by the amount s which we shall call the shift.

In 1950 K. Artmann [1] showed that for Z=0 and Z=∞ the boundary of the shadow of a circular cylinder was shifted by the amounts

$$s = -.20(2\pi ka)^{-2/3}a = -.68(ka)^{-2/3}a \qquad Z = 0\left[\frac{\partial u(a,\theta)}{\partial r} = 0\right] \tag{28}$$

$$s = .39(2\pi ka)^{-2/3}a = 1.33(ka)^{-2/3}a \qquad Z = \infty\left[u(a,\theta) = 0\right] \tag{29}$$

In 1954 S.O. Rice [2] showed that for Z=0 and Z=∞ the boundary of the shadow cast by a parabolic cylinder was also shifted. In terms of the radius of curvature, a, of the parabola at the point of tangency of the geometrical shadow, the shift was found to be

$$s = -.346(2\pi ka)^{-2/3}a = -1.18(ka)^{-2/3}a \qquad Z = 0 \tag{30}$$

$$s = .399(2\pi ka)^{-2/3}a = 1.36(ka)^{-2/3}a \qquad Z = \infty \tag{31}$$

The process of shadow boundary formation is local in character. Therefore it is to be expected that the leading term in the expression for the shift should depend only upon the radius of curvature of the shadow forming cylinder at the point of tangency of the geometrical shadow boundary. The results (29) and (31) tend to confirm this expectation while (28) and (30) seem to contradict it. Let us compare these results with our result (27). When Z=0 and Z=∞, S.I. Rubinow and T.T. Wu [4] and T.T. Wu [5] have shown that $C_I = \sqrt{3}\, C_R$ and have computed $C_R(0) = -0.432120$ and $C_R(\infty) = 0.498077$. Using these results in (27) we obtain for the shifts in these cases

$$s = -1.180574\ (ka)^{-2/3}a \qquad Z = 0\left[\frac{\partial u(a,\theta)}{\partial r} = 0\right] \tag{32}$$

$$s = 1.360771\ (ka)^{-2/3}a \qquad Z = \infty\left[u(a,\theta) = 0\right] \tag{33}$$

We see that our results (32) and (33) for the circular cylinder agree with those for the parabolic cylinder given in (30) and (31). Thus the discrepancy was due to an inaccurate determination by Artmann of the constant in (28).

This agreement confirms our expectation. It also leads us to conjecture that the shift of the boundary of the shadow cast by any object is given by (27), as described in the Introduction. To check it, let us consider a plane wave incident obliquely upon a cylinder of arbitrary cross section. Let $\pi/2 - \gamma$ denote the angle between the incident rays and the generators of the cylinder. Let a' be the radius of curvature of the normal cross section of the cylinder at one of its two points of tangency with the shadow boundary. Then the radius of curvature a of the cylinder surface in the plane containing the incident tangent ray and the normal to the cylinder, is $a = a' \sec^2 \gamma$. Thus (27) yields for the shift

$$s = (C_R + C_I)(ka')^{-2/3} a' (\sec \gamma)^{2/3}$$

where

$$C = C\left[(ka')^{1/3} Z (\sec \gamma)^{2/3}\right]$$

Another expression for s can be obtained from the solution $v(x,y,z)$ of the boundary value problem for oblique incidence. It is known that v can be related to $u(x,y)$, the solution for normal incidence, which we have previously considered. The relation, which involves replacing k by $k \cos \gamma$ and z by $z \sec \gamma$, is

$$v = \exp\left[ikz \sin \gamma\right] u(x,y,k \cos \gamma,\ z \sec \gamma).$$

If (27) is used for the shift of the shadow boundary of $u(x,y,k \cos \gamma,\ 2 \sec \gamma)$, it yields exactly the result given above. This agreement shows that our conjecture is consistent, i.e., it is correct for oblique incidence if it is correct for normal incidence. In those cases for which we know it to be correct at normal incidence (circular and parabolic cylinders) it is therefore also correct for oblique incidence.

5. The scattering cross section of the cylinder.

When ka is large, the cross section σ is asymptotically given by the first term in (10) since the sum is relatively negligible for the reasons outlined previously. That term is an integral which can be evaluated by splitting it into σ_1 and σ_2 just as the integral for $G(r,\theta)$ was split in (11) and (12).

$$\sigma_1 = \frac{4}{k} \text{ R.P. } \int_0^{ka} d\nu = 4a \tag{34}$$

$$\sigma_2 = \frac{4}{k} \text{ R.P.} \left\{ \int_{-\infty}^{ka} \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)} d\nu + \int_{ka}^{\infty} \left[1 + \frac{\Omega H_\nu^{(2)}(ka)}{\Omega H_\nu^{(1)}(ka)} \right] d\nu \right\} \tag{35}$$

Then

$$\sigma \sim \sigma_1 + \sigma_2 \tag{36}$$

We note that $\sigma_1 = 4a$ is twice the diameter of the cylinder, so it is just the geometrical optics scattering cross section. Thus σ_2 is the correction to it due to the finiteness of ka.

The integrals in (35) can be evaluated in the same way as were those for $G_2(r,\theta)$ in (12). The result is similar to (22), and (35) becomes

$$\sigma_2 = \frac{4}{k}(ka)^{1/3} C_R \tag{37}$$

Here C_R is the real part of $C\left[(ka)^{1/3} Z\right]$ which is given by (23). Now (36) becomes

$$\sigma \sim 4a\left[1 + C_R (ka)^{-2/3}\right] \tag{38}$$

For $Z=0$ and $Z=\infty$ this result coincides with that of S.I. Rubinow and T.T. Wu [4].

6. Relation of the shadow boundary shift to the cross section.

Upon comparing (27) with (38) we find

$$\sigma \sim 4\left[a + \frac{C_R}{C_R + C_I}\, s\right] \tag{39}$$

Equation (39) shows that the scattering cross section is the same as the geometrical optics cross section of a cylinder with the radius $a + \frac{C_R}{C_R+C_I}\, s$. For Z=0 and Z=∞, $C_R(C_R+C_I)^{-1} = (1 + \sqrt{3})^{-1}$.

Let us now consider a non-circular cylinder with two shadow boundaries and two shifts s_1 and s_2. We expect each shift to be given by (27) with the appropriate radius of curvature a_1 or a_2 of the cylinder at its point of tangency with the corresponding shadow boundary. Let w denote the width of the shadow according to geometrical optics. Then by analogy with (39), and based upon the physics of shadow formation, we expect the scattering cross section of the cylinder to be

$$\sigma \sim 2\left[w + \left(\frac{C_R s}{C_R + C_I}\right)_1 + \left(\frac{C_R s}{C_R + C_I}\right)_2\right]$$

$$\sim 2\left[w + k^{-2/3}(C_R a^{1/3})_1 + k^{-2/3}(C_R a^{1/3})_2\right] . \tag{40}$$

In case Z=0 or Z=∞, C_R is independent of a, and (40) simplifies slightly. This simpler formula was suggested previously on the basis of similar considerations [6] .

Next we consider the shadow of a three dimensional object. Along each ray of the geometrical shadow boundary we expect a shift normal to this boundary. The shift should still be given by (27) with a the radius of curvature of the object's surface at the point of tangency of the ray, in the plane containing this ray and the surface normal. Let τ denote arclength along a normal cross

section of the shadow boundary. Then if A is the cross sectional area of the geometrical shadow we should have

$$\sigma \sim 2\left[A + \oint \frac{C_R s}{C_R + C_I}\, d\tau\right] = 2\left[A + k^{-2/3} \oint C_R a^{1/3} d\tau\right] \tag{41}$$

This expression as well as (40) should also apply when Z is not constant. When $Z=0$ or $Z=\infty$ and C_R is independent of a, (41) becomes

$$\sigma \sim 2\left[A + C_R k^{-2/3} \oint a^{1/3} d\tau\right]. \tag{42}$$

As an example of the use of (41) let us determine σ for a body of revolution with the incident plane wave traveling along its axis. Let ρ denote the maximum radius of the body. Then (41) becomes

$$\sigma \sim 2\pi\rho^2 + 4\pi C_R k^{-2/3} a^{1/3} \rho \tag{43}$$

For a sphere, $\rho=a$, and (43) reduces to

$$\sigma \sim 2\pi a^2\left[1 + 2C_R (ka)^{-2/3}\right] \tag{44}$$

In the special cases $Z=0$ and $Z=\infty$ the result (44) for the sphere agrees with previously known results [4,5]. This agreement is a verification of the assumptions on which (41) is based.

7. The electromagnetic case.

Let us now consider an elliptically polarized monochromatic plane electromagnetic wave incident normally upon a perfectly conducting cylinder of arbitrary cross section. We assume that the incident electric field $\vec{E}^{inc}$ is given by

$$\vec{E}^{inc} = (\vec{k} \cos\beta + \vec{j}\, e^{i\delta} \sin\beta) e^{ikx} \tag{45}$$

Here $\vec{j}$ and $\vec{k}$ are unit vectors along the y and z axes, respectively, while β and δ are real constants. The cylinder generators are parallel to the z axis and the time factor $e^{-i\omega t}$ is omitted. The propagation constant is $k = \omega\sqrt{\epsilon\mu}$ where ϵ and μ are the dielectric constant and permeability of the medium. Then the total fields, electric and magnetic, consisting of the incident and scattered fields, can be written as

$$\vec{E} = \vec{k} u_\infty \cos\beta - ik^{-1} e^{i\delta} \sin\beta\, \vec{k} \times \nabla u_o \tag{46}$$

$$\vec{H} = \sqrt{\frac{\epsilon}{\mu}} \left(\vec{k} u_o\, e^{i\delta} \sin\beta + ik^{-1} \cos\beta\, \vec{k} \times \nabla u_\infty \right) \tag{47}$$

In these equations u_o and u_∞ are the fields which result when the scalar plane wave e^{ikx} is incident upon a hard or soft cylinder with the same cross section as the given one.

On and near the upper boundary of the shadow of a circular cylinder, u_o and u_∞ are given by (26). This equation shows that in both cases $\nabla u \sim iku\vec{i}$, where $\vec{i}$ is the unit vector along the x axis. Upon using this result in (46) and (47) we obtain for the fields on and near the shadow boundary

$$\vec{E} \sim \vec{k}u_{\infty} \cos \beta + \vec{j}u_{o}e^{i\delta} \sin \beta \tag{48}$$

$$\vec{H} \sim \sqrt{\frac{\epsilon}{\mu}} \, (\vec{k}u_{o}e^{i\delta} \sin \beta - \vec{j}u_{\infty} \cos \beta) \tag{49}$$

From (48) and (49) we find that the time averaged electromagnetic energy density is given by

$$\tfrac{1}{4}(\epsilon|E|^2 + \mu|H|^2) \sim \tfrac{\epsilon}{2}(|u_{\infty}|^2 \cos^2\beta + |u_{o}|^2 \sin^2\beta) \tag{50}$$

Upon inserting (26) into (50) we obtain

$$\tfrac{1}{4}(\epsilon|E|^2 + \mu|H|^2) \sim \frac{\epsilon}{2} \left\{\frac{1}{4} - (4\pi kx)^{-1/3}\left[k(a-y) + (ka)^{1/3} \left(C_R(\infty) + C_I(\infty)\right) \cos^2\beta \right.\right.$$
$$\left.\left. + (ka)^{1/3} \left(C_R(0) + C_I(0)\right) \sin^2\beta\right]\right\} \tag{51}$$

At the geometrical shadow boundary $y=a$ the right side of (51) approaches $\epsilon/8$ as ka becomes infinite. Let us define the shadow boundary for finite values of ka as the place where the energy density has this same value $(\epsilon/8)$. Then (51) shows that the shadow boundary is the straight line $y=a+s_{EM}$. Here s_{EM}, the shift in the electromagnetic case, is given by

$$s_{EM} = \left\{\left[C_R(\infty) + C_I(\infty)\right]\cos^2\beta + \left[C_R(0) + C_I(0)\right]\sin^2\beta\right\} (ka)^{-2/3} a$$

$$= s_{\infty} \cos^2\beta + s_{o} \sin^2\beta \tag{52}$$

The shifts s_{∞} and s_{o} in the soft and hard cases, respectively, are given by (27). Since $C_I = \sqrt{3}\, C_R$ for $Z=0$ and $Z=\infty$, (52) can be written as

$$s_{EM} = \left[-1.180574 \cos^2\beta + 1.360771 \sin^2\beta\right](ka)^{-2/3}a \qquad (53)$$

Just as in the scalar case, we conjecture that the shift along each ray of the boundary of the shadow cast by an arbitrary object is given by (52). The radius a is defined as in the scalar case. The angle β is given in terms of the tangential and normal components, E_t^{inc} and E_n^{inc}, of the incident field at the point of tangency of the ray and the surface of the object, by

$$\tan\beta = |E_n^{inc}|/|E_t^{inc}| \qquad (54)$$

This conjecture can be shown to be consistent for cylinders of arbitrary cross section, as in the scalar case.

The relations (48) and (49) also hold far from the cylinder in the forward direction, i.e. in the direction of the positive x-axis. This is so because in that direction $u \sim e^{ikx} + Br^{-1/2}e^{ikr}$ where B is some constant, and $r \sim x$. Thus $\nabla u \sim ik u\vec{i}$ and this, when used in (46) and (47), yields (48) and (49). Upon applying the electromagnetic cross section theorem to (48) and (49) we obtain for the total scattering cross section in the electromagnetic case

$$\sigma_{EM} = \sigma_\infty \cos^2\beta + \sigma_o \sin^2\beta \qquad (55)$$

Here σ_o and σ_∞ are the cross sections for the scalar hard and soft cases, respectively. Upon using (38) for σ_o and σ_∞ in (55) we obtain the result

$$\begin{aligned}\sigma_{EM} &\sim 4a \left\{1 + \left[C_R(\infty)\cos^2\beta + C_R(0)\sin^2\beta\right](ka)^{-2/3}\right\} \\ &\sim 4a \left\{1 + \left[0.498077 \cos^2\beta - 0.432120 \sin^2\beta\right](ka)^{-2/3}\right\}\end{aligned} \qquad (56)$$

By comparing (56) with (52) or (53) we find that σ_{EM} and s_{EM} are related by

$$\sigma_{EM} \sim 4\left[a + (1+\sqrt{3}\ )^{-1} s_{EM}\right] = 4a + 3.414102\ s_{EM} \tag{57}$$

As in the scalar case, this relation suggests that for an arbitrary cylinder the total electromagnetic scattering cross section per unit length is

$$\begin{aligned} \sigma_{EM} &\sim 2\ \left\{w + (1+\sqrt{3})^{-1}\left[(s_{EM})_1 + (s_{EM})_2\right]\right\} \\ &\quad 2\ \left\{w + \left[0.498077\ \cos^2\beta - 0.432120\ \sin^2\beta\right]k^{-2/3}(a_1^{1/3} + a_2^{1/3})\right\} \end{aligned} \tag{58}$$

Here w is the width of the geometrical optics shadow while a_1 and a_2 are the two radii of curvature of the cylinder's cross sectional boundary at its points of tangency with the shadow boundary. Similarly for a three dimensional object the total cross section should be

$$\begin{aligned} \sigma_{EM} &\sim 2\ \left\{A + (1+\sqrt{3})^{-1} \oint s_{EM} d\tau\right\} \\ &\sim 2\ \left\{A + k^{-2/3}\left[0.498077 \oint a^{1/3}\cos^2\beta d\tau - 0.432120 \oint a^{1/3}\sin^2\beta d\tau\right]\right\} \end{aligned} \tag{59}$$

Let us apply (59) to an object of revolution with a plane wave incident along the axis of revolution. Let ρ be the maximum radius of the object. Then (59) yields

$$\begin{aligned} \sigma_{EM} &\sim 2\pi\rho^2 + 2\pi\left[C_R(\infty) + C_R(0)\right]k^{-2/3}a^{1/3}\rho \\ &\sim 2\pi\rho^2 + 2\pi\left[0.065957\right]k^{-2/3}a^{1/3}\rho \end{aligned} \tag{60}$$

Upon comparing (60) with (43) we find that for axial incidence on an object of revolution

$$\sigma_{EM} \quad \frac{1}{2}(\sigma_o + \sigma_\infty) \tag{61}$$

For a sphere $\rho = a$ and (60) becomes

$$\sigma_{EM} \sim 2\pi a^2 \left[1 + 0.65957(ka)^{-2/3}\right] \tag{62}$$

This result agrees with the known result [4,5] in this case, which is a verification of our conjecture.

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